

May, 2000

OU-HET 347

hep-ph/0005267

# Renormalization group effect and a democratic-type neutrino mass matrix

Takahiro Miura<sup>\*</sup>, Tetsuo Shindou<sup>†</sup>, Eiichi Takasugi<sup>‡</sup>

*Department of Physics, Osaka University*

*Toyonaka, Osaka 560-0043, Japan*

Masaki Yoshimura<sup>§</sup>

*Department of Physics, Ritsumeikan University*

*Kusatsu, Shiga 525-8577, Japan*

## Abstract

In our previous paper, we proposed the democratic-type neutrino mass matrix which gives interesting predictions,  $\theta_{23} = -\frac{\pi}{4}$ ,  $|\tan \theta_{12}| = \sqrt{2 - 3 \sin^2 \theta_{13}}$  and  $\delta = \frac{\pi}{2}$ , where  $\theta_{ij}$  is the mixing angle between mass eigenstates  $\nu_i$  and  $\nu_j$ , and  $\delta$  is the CP violation angle in the standard parameterization of mixing matrix. In this paper, we examined how predictions behave at  $m_Z$  by assuming that they are given at the right-handed neutrino mass scale,  $M_R$ .

---

<sup>\*</sup>e-mail address: miura@het.phys.sci.osaka-u.ac.jp

<sup>†</sup>e-mail address: shindou@het.phys.sci.osaka-u.ac.jp

<sup>‡</sup>e-mail address: takasugi@het.phys.sci.osaka-u.ac.jp

<sup>§</sup>e-mail address: myv20012@se.ritsumei.ac.jp

# 1 Introduction

In our recent papers[1],[2], we proposed the democratic-type mass matrix which contains six real parameters and found that this mass matrix predicts

$$\theta_{23} = -\frac{\pi}{4}, \quad \delta = \frac{\pi}{2}, \quad (1)$$

where  $\theta_{23}$  and  $\delta$  are the mixing angle between the mass eigenstates,  $\nu_2$  and  $\nu_3$ , and the CP violation phase, in the parameterization of the mixing matrix given in the particle data group[3] (see the matrix given in the Appendix A).

If we take the CHOOZ bound[4],  $|V_{13}| < 0.16$  or  $|\sin \theta_{13}| < 0.16$ , we find almost maximum atmospheric neutrino mixing,

$$\sin^2 2\theta_{atm} = 4|V_{23}|^2(1 - |V_{23}|^2) = 1 - \sin^4 \theta_{13} > 0.999, \quad (2)$$

where  $V$  is the neutrino mixing matrix. If the experimental data turns out to show that  $\sin^2 2\theta_{atm}$  is really close to unity, our model will become a good candidate. Another special feature of the model is the prediction of the value of the CP violation phase. Both Dirac CP phase ( $\delta$ ) and Majorana CP phases[5] are predicted[1]. In particular, the maximal value of the CP violation phase  $\delta$  is predicted. Our prediction gives the great encouragement for experiments to measure the CP violation in the oscillation processes[6] in the near future. The theoretical study has become an urgent topic.

In Ref.2, we made a further investigation on the democratic-type mass matrix. We constructed  $Z_3$  invariant Lagrangian with two or three up-type Higgs doublets and derived the democratic-type mass matrix by using the see-saw mechanism. We also considered one up-type Higgs model. By considering the  $Z_3$  symmetric Lagrangian together with the  $Z_2$  invariant  $Z_3$  breaking terms, we found the further prediction,

$$|\tan \theta_{12}| = \sqrt{2 - 3 \sin^2 \theta_{13}}, \quad (3)$$

which we shall explain in the next section. By using the CHOOZ bound, this relation leads to

$$0.87 < \sin^2 2\theta_{sol} = 4|V_{11}|^2|V_{12}|^2 < \frac{8}{9}. \quad (4)$$

In Refs.1 and 2, we assumed that the above predictions are valid at the weak scale  $m_Z$ , although the neutrino mass matrix is assumed to be defined at the right-handed neutrino mass scale  $M_R$ . The stability of mixing angles under the change of energy scale has been discussed[7-10]. According to their result, in many occasions, the predictions at  $m_Z$  are essentially the same as those at  $M_R$ . In some special cases where  $m_1 \simeq m_2$ , the prediction of  $\sin^2 2\theta_{sol}$  becomes unstable. That is, the predicted large value of  $\sin^2 2\theta_{sol}$  at  $M_R$  becomes the small value at  $m_Z$ .

The purpose of this paper is to examine the stability of our predictions. In particular, we are interested in the possibility that the large solar neutrino mixing at  $M_R$  becomes small to be consistent with the small angle MSW solution at  $m_Z$ . We found that the angle can become small, but unfortunately this possibility does not realize the small angle MSW solution.

In Sec.2, we briefly explain our model. In Sec.3, we analytically examine the renormalization effect on the neutrino mass matrix and the effect to our predictions. The numerical analysis to supplement the discussions in Sec.3 is given in Sec.4. In Sec.5, the summary is given.

## 2 The model

We consider the following dimension five Lagrangian in the mass eigenstate basis of charged leptons[2]

$$\mathcal{L}_Y = -(m_1^0 + \tilde{m}_1) \overline{(\Psi_1)^C} \Psi_1 \frac{H_u H_u}{u_u^2} - 2\tilde{m}_1 \overline{(\Psi_2)^C} \Psi_3 \frac{H_u H_u}{u_u^2}, \quad (5)$$

where  $\tilde{m}_1$  and  $m_1^0$  are real parameters, and  $u_u$  is the vacuum expectation value of the neutral component of the doublet Higgs  $H_u$ . This Lagrangian is invariant under the  $Z_3$  transformation

$$\Psi_1 \rightarrow \omega \Psi_1, \quad \Psi_2 \rightarrow \omega^2 \Psi_2, \quad \Psi_3 \rightarrow \Psi_3, \quad H_u \rightarrow \omega^2 H_u, \quad (6)$$

where, the irreducible representation  $\Psi_i$  ( $i = 1, 2, 3$ ) are defined by

$$\begin{aligned}\Psi_1 &= \frac{1}{\sqrt{3}}(\ell_e + \omega^2 \ell_\mu + \omega \ell_\tau), \\ \Psi_2 &= \frac{1}{\sqrt{3}}(\ell_e + \omega \ell_\mu + \omega^2 \ell_\tau), \\ \Psi_3 &= \frac{1}{\sqrt{3}}(\ell_e + \ell_\mu + \ell_\tau).\end{aligned}\tag{7}$$

The  $Z_3$  transformation for  $\Psi_i$  is induced by the cyclic permutation among  $\ell_i$ , which are the left-handed lepton doublets defined by  $\ell_e = (\nu_{eL}, e_L)^T$  and so on.

Then, we introduce the  $Z_3$  symmetry breaking term, but it preserves the  $Z_2$  symmetry

$$\Psi_1 \rightarrow -\Psi_1, \tag{8}$$

and all other fields are unchanged. Now, we find

$$\mathcal{L}_{SB} = -m_2^0 \overline{(\Psi_2)^C} \Psi_2 \frac{H_u H_u}{u_u^2} - m_3^0 \overline{(\Psi_3)^C} \Psi_3 \frac{H_u H_u}{u_u^2}. \tag{9}$$

After  $H_u$  acquires the vacuum expectation value, the neutrino mass term is derived. In the flavor basis,  $(\nu_e, \nu_\mu, \nu_\tau)$ , the mass matrix is given in a democratic-type form[2],

$$\begin{aligned}m_\nu(M_R) &= \frac{m_1^0}{3} \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{pmatrix} + \tilde{m}_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \\ &\quad + \frac{m_2^0}{3} \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} + \frac{m_3^0}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},\end{aligned}\tag{10}$$

where  $\omega$  is the element of  $Z_3$  symmetry and we take  $\omega = \exp(i2\pi/3)$ , i.e.,  $\omega^3 = 1$ . We consider that this mass matrix is given at the right-handed mass scale  $M_R$ .

The unitary matrix  $V_2$  which diagonalizes  $m_\nu(M_R)$  is derived in the Appendix A and the result is

$$V_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}}c' & i\sqrt{\frac{2}{3}}s' \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}(c' + i\sqrt{3}s') & -\frac{1}{\sqrt{6}}(\sqrt{3}c' + is') \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}}(c' - i\sqrt{3}s') & \frac{1}{\sqrt{6}}(\sqrt{3}c' - is') \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{11}$$

where  $c' = \cos \theta'$  and  $s' = \sin \theta'$  and

$$\tan \theta' = \frac{\Delta_-}{\tilde{m}_1 + \sqrt{\tilde{m}_1^2 + \Delta_-^2}} , \quad (12)$$

with  $\Delta_- = (m_3^0 - m_2^0)/2$ .

It should be noted that predictions in Eqs.(1) and (3) are derived from  $V_2$ . The phase in  $\text{diag}(1, -1, i)$  represents the Majorana phase, while the phases in  $\text{diag}(1, \omega, \omega^2)$  are the irrelevant phases which are absorbed by the charged lepton fields.

From our later analysis, there is essentially no effect to  $V_{13}$ . As a result, we can impose the CHOOZ bound,  $|V_{13}| < 0.16$  at  $m_Z$ . We find

$$|s'| < 0.2 . \quad (13)$$

We define the mass eigenstate neutrinos at  $M_R$  as  $(\nu_1^R, \nu_2^R, \nu_3^R)$  and their masses are

$$\begin{aligned} m_1^R &= m_1^0 + \tilde{m}_1 , \\ m_2^R &= m_2^0 + \Delta_- + \sqrt{\tilde{m}_1^2 + \Delta_-^2} , \\ m_3^R &= m_2^0 + \Delta_- - \sqrt{\tilde{m}_1^2 + \Delta_-^2} . \end{aligned} \quad (14)$$

We take the convention,  $\tilde{m}_1 > 0$ . Since  $m_2^0$  and  $m_3^0$  are parameters for the symmetry breaking terms, we expect that  $\tilde{m}_1 \gg |m_2^0|, |m_3^0|$ . Then, we find  $m_2^R > 0$  and  $m_3^R < 0$ . The parameter  $\tilde{m}_1$  controls the overall normalization neutrino masses, and  $m_1^0$  and  $m_2^0$  (or  $m_3^0$ ) control the mass of  $m_1^R$  and the mass splitting between  $m_2^R$  and  $m_3^R$ , while the parameter  $\Delta_- = (m_3^0 - m_2^0)/2$  does the size of  $V_{13}$ .

### 3 The renormalization group analysis

We consider the renormalization group effect on the dimension five interaction in Eqs.(5) and (9) in the MSSM model. The general feature of the stability of mixing angles has been extensively discussed[7-10]. Here, we take the special mass matrix, the democratic-type mass matrix and examine the stability in detail.

(3-1) Neutrino mass matrix at  $m_Z$

In the basis where charged lepton mass matrix is diagonal and thus the Yukawa coupling matrix which induces masses of charged leptons is diagonal, the neutrino mass matrices at  $M_R$  and  $m_Z$  are related as[7]

$$m_\nu(M_R) \simeq A \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{I_\mu}{I_e}} & 0 \\ 0 & 0 & \sqrt{\frac{I_\tau}{I_e}} \end{pmatrix} m_\nu(m_Z) \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{I_\mu}{I_e}} & 0 \\ 0 & 0 & \sqrt{\frac{I_\tau}{I_e}} \end{pmatrix}, \quad (15)$$

where

$$I_i = \exp\left(\frac{1}{8\pi^2} \int_{\ln(m_Z)}^{\ln(M_R)} y_i^2 dt\right) \quad (i = e, \mu, \tau), \quad (16)$$

with the Yukawa coupling for charged leptons  $y_i$  and  $A = (I_e/I_\tau)(m_{\nu 33}(M_R)/m_{\nu 33}(m_Z))$ .

After absorbing  $A$  into the overall normalization of parameters in  $m_\nu(M_R)$  and by using the approximation,

$$\sqrt{\frac{I_\mu}{I_\tau}} \simeq \sqrt{\frac{I_e}{I_\tau}} \simeq \frac{1}{\sqrt{I_\tau}}, \quad (17)$$

we obtain

$$m_\nu(m_Z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix} m_\nu(M_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix}, \quad (18)$$

where

$$\alpha \equiv 1/\sqrt{I_\tau} = \left(\frac{m_Z}{M_R}\right)^{\frac{1}{8\pi^2}(1+\tan^2\beta)(m_\tau/v)^2} < 1. \quad (19)$$

Here we neglect the radiative correction on  $y_\tau$ , and  $m_\tau$  is the  $\tau$  lepton mass,  $v^2 = u_u^2 + u_d^2$  and  $\tan\beta = u_u/u_d$  with  $u_i$  being the vacuum expectation value of MSSM Higgs doublet  $< H_i >$  ( $i = u, d$ ).

Now we define the small parameter  $\epsilon = 1 - \alpha$ . In order to estimate the value of  $\epsilon$ , we consider the right-handed mass scale  $M_R$  and the region of  $\tan\beta$  as

$$M_R = 10^{13}(\text{GeV}), \quad 2 < \tan\beta < 60. \quad (20)$$

Then, with  $m_Z = 91.187(\text{GeV})$ ,  $m_\tau = 1.777(\text{GeV})$  and  $v = 245.4(\text{GeV})$ , we find

$$8 \times 10^{-5} < \epsilon < 6 \times 10^{-2} . \quad (21)$$

(3-2) The diagonalization

By transforming  $m_\nu(m_Z)$  in Eq.(18) by  $V_2$ , we find

$$\begin{aligned} \tilde{m}_\nu &\equiv V_2^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix} V_2^* \left( V_2^T m_\nu(M_R) V_2 \right) V_2^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix} V_2 \\ &= (1 - \epsilon S^T) D_\nu^R (1 - \epsilon S) , \end{aligned} \quad (22)$$

where  $D_\nu^R = V_2^T m_\nu(M_R) V_2 = \text{diag}(m_1^R, m_2^R, m_3^R)$  and

$$S = V_2^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_2 = \frac{1}{3} \begin{pmatrix} 1 & -\frac{1}{\sqrt{2}} a e^{-i\phi_1} & i\sqrt{\frac{3}{2}} b e^{-i\phi_2} \\ -\frac{1}{\sqrt{2}} a e^{i\phi_1} & \frac{1}{2} a^2 & -i\frac{\sqrt{3}}{2} a b e^{i(\phi_1 - \phi_2)} \\ -i\sqrt{\frac{3}{2}} b e^{i\phi_2} & i\frac{\sqrt{3}}{2} a b e^{-i(\phi_1 - \phi_2)} & \frac{3}{2} b^2 \end{pmatrix} , \quad (23)$$

where  $s'$  and  $c'$  are given in Eq.(12), and  $a$ ,  $b$  and phases  $\phi_i$  are defined by

$$\begin{aligned} a &= \sqrt{1 + 2s'^2} , & b &= \sqrt{1 - \frac{2}{3}s'^2} , \\ \tan \phi_1 &= \sqrt{3} \tan \theta' , & \tan \phi_2 &= \frac{1}{\sqrt{3}} \tan \theta' . \end{aligned} \quad (24)$$

By keeping  $\epsilon$  up to the first order, we find

$$\tilde{m}_\nu \simeq \begin{pmatrix} (1 - \frac{2}{3}\epsilon)m_1^R & \frac{1}{3\sqrt{2}}\epsilon a(m_1^R e^{-i\phi_1} + m_2^R e^{i\phi_1}) & -i\epsilon p \\ \frac{1}{3\sqrt{2}}\epsilon a(m_1^R e^{-i\phi_1} + m_2^R e^{i\phi_1}) & (1 - \frac{1}{3}a^2\epsilon)m_2^R & i\epsilon q \\ -i\epsilon p & i\epsilon q & (1 - b^2\epsilon)m_3^R \end{pmatrix} , \quad (25)$$

where

$$\begin{aligned} p &= \frac{1}{\sqrt{6}} b(m_1^R e^{-i\phi_2} - m_3^R e^{i\phi_2}) , \\ q &= \frac{1}{2\sqrt{3}} a b(m_2^R e^{i(\phi_1 - \phi_2)} - m_3^R e^{-i(\phi_1 - \phi_2)}) . \end{aligned} \quad (26)$$

(3-3) The general discussion on the stability

Hereafter, we do not discuss the fully degenerate case,  $|m_1^R| \simeq |m_2^R| \simeq |m_3^R|$ , because this case is quite unstable and it is hard to have the definite predictions. Therefore, we focus our discussions on hierarchical cases; (a)  $|m_3^R| \gg |m_2^R| \gg |m_1^R|$  or  $|m_3^R| \gg |m_1^R| \gg |m_2^R|$  and (b)  $|m_1^R| \simeq |m_2^R| \gg |m_3^R|$  or  $|m_3^R| \gg |m_2^R| \simeq |m_1^R|$ .

The case (a): The fully hierarchical case

With the use of the analogy of the analysis by Haba et al., we expect that all mixing angles and the CP violation phase are essentially unchanged by the scale change from  $M_R$  to  $m_Z$ . This may be simply understood by the consideration that the see-saw mechanism can be used to evaluate the mixings and the neutrino masses, and thus the effect is suppressed by the order of  $\epsilon$ . We checked this result by the numerical computations also.

The case (b): The hierarchical case with  $|m_1^R| \simeq |m_2^R|$

The situation is slightly complicated in comparison with the case (a), because of the degeneracy  $|m_1^R| \simeq |m_2^R|$ . Firstly, we notice that the off diagonal terms are much small than  $(\tilde{m}_\nu)_{33}$ , or  $|(\tilde{m}_\nu)_{11}| \simeq |(\tilde{m}_\nu)_{22}|$ . Therefore, we can use the see-saw calculation between  $(\nu_1^R, \nu_2^R)$  and  $\nu_3^R$ , where  $\nu_i^R$  are mass eigenstates at  $M_R$ . That is, we can safely neglect the contributions from  $p$  and  $q$  terms in the matrix and thus we do not need to consider the mixing between  $(\nu_1^R, \nu_2^R)$  and  $\nu_3^R$ .

Now, the element  $V_{i3}$  and  $V_{3i}$  ( $i = 1, 2, 3$ ) at  $M_R$  is still valid at  $m_Z$ . That is,  $V_{i3} = (V_2)_{i3}$  and  $V_{3i} = (V_2)_{3i}$ . As a result, the prediction of  $\sin^2 2\theta_{atm} > 0.999$  in Eq.(2) and the CHOOZ constraint,  $|s_{13}| < 0.16$  are valid at  $m_Z$ .

The situation changes depending on the relative sign between  $m_1^R$  and  $m_2^R$ .

(b-1) The case where  $m_1^R < 0$  and  $m_2^R > 0$

We denote the submatrix for  $(\nu_1^R, \nu_2^R)$  as  $\tilde{m}'_\nu$  with the approximation  $a \simeq 1$  because  $s'^2 < 0.04$  is small,

$$\begin{aligned} \tilde{m}'_\nu &\simeq \begin{pmatrix} -(1 + \Delta - \frac{2}{3}\epsilon) & i\frac{\sqrt{2}}{3}\epsilon \sin \phi_1 \\ i\frac{\sqrt{2}}{3}\epsilon \sin \phi_1 & 1 - \frac{1}{3}\epsilon \end{pmatrix} m_2^R \\ &= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} -(1 + \Delta - \frac{2}{3}\epsilon) & \frac{\sqrt{2}}{3}\epsilon \sin \phi_1 \\ \frac{\sqrt{2}}{3}\epsilon \sin \phi_1 & -(1 - \frac{1}{3}\epsilon) \end{pmatrix} m_2^R \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \end{aligned} \quad (27)$$



where we defined

$$\Delta = \frac{|m_1^R| - m_2^R}{m_2^R} . \quad (28)$$

The matrix  $\tilde{m}'_\nu$  is diagonalized by

$$\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} , \quad (29)$$

where  $c = \cos \theta$  and  $s = \sin \theta$  and

$$\begin{aligned} \tan \theta &= \frac{\pm \frac{2\sqrt{2}}{3}\epsilon \sin \phi_1}{\frac{1}{3}\epsilon - \Delta + \sqrt{(\frac{1}{3}\epsilon - \Delta)^2 + \frac{8}{9}\epsilon^2 \sin^2 \phi_1}} , \\ ||m_1| - m_2| &= m_2^R \sqrt{(\frac{1}{3}\epsilon - \Delta)^2 + \frac{8}{9}\epsilon^2 \sin^2 \phi_1} , \end{aligned} \quad (30)$$

and  $m_2 \simeq m_2^R$ .

The mixing matrix at  $m_Z$  is now obtained by multiplying this matrix to  $V_2$  in Eq.(11). By looking at the structure of  $V_2$ , we find

$$\begin{aligned} V_{11} &= \frac{1}{\sqrt{3}}(c - i\sqrt{2}c's) , \\ V_{12} &= \frac{1}{\sqrt{3}}(-s - i\sqrt{2}c'c) , \end{aligned} \quad (31)$$

aside from the irrelevant phases. By neglecting the small  $s'^2 < 0.04$ , we have  $c' = 1$  and thus we find

$$\sin^2 2\theta_{sol} \simeq \frac{8}{9}(1 + s^2)(1 - \frac{s^2}{2}) , \quad (32)$$

which takes a value from 8/9 to 1 independent of the mixing angle  $\theta$ . This is due to the phase matrix  $\text{diag}(1, -i)$  in Eq.(29).

By the transformation of the matrix in Eq.(29), the CP violation phase  $\delta$  changes, due to the phase matrix  $\text{diag}(1, -i)$ . The effect is examined by considering the Jarlskog parameter which takes the value as

$$|J_{CP}| \equiv |\text{Im}(V_{11}V_{12}^*V_{21}^*V_{22})| = \frac{1}{3\sqrt{3}}|s'c'(c^2 - s^2)| , \quad (33)$$

and we find

$$|\sin \delta| = \frac{|\cos 2\theta|}{\sqrt{1 + \frac{1}{8} \sin^2 2\theta \left( \frac{\cos 2\theta'}{\cos \theta'} \right)^2}}. \quad (34)$$

It should be noted that  $\theta = 0$  at  $M_R$  so that  $|\sin \delta| = 1$ . Now we examine the value at  $m_Z$  from Eq.(34). The angle  $\theta$  depends on  $\Delta$  which is defined in Eq.(28), as we can see in Eq.(30). For  $\Delta \gg \epsilon$  or  $\Delta < 0$ ,  $|\tan \theta| \gg 1$  or  $|\tan \theta| \ll 1$ . Therefore,  $|\sin \delta| \sim 1$  is realized from Eq.(34). In special cases where  $\Delta \simeq \epsilon/3$ ,  $\sin \delta$  can become small at  $m_Z$ . In particular, for  $\Delta = \epsilon/3$ , we find  $\tan \theta = \pm 1$  and thus we find  $\sin \delta = 0$ .

Finally, we find

$$\Delta_{sol}^2 = |m_2^2 - m_1^2| \simeq 2m_2^2 \sqrt{\left(\frac{1}{3}\epsilon - \Delta\right)^2 + \frac{8}{9}\epsilon^2 \sin^2 \phi_1}, \quad (35)$$

which depends on  $m_2$  and  $\Delta$ . Therefore, we can reproduce all three mass squared differences for the large angle MSW, the LOW mass and the Just so (Vacuum) solutions. For example, when  $|\Delta| \gg \epsilon$ , we find  $\Delta_{sol}^2 \simeq 2m_2^2 \Delta \simeq (\Delta_{sol}^2)_{M_R}$ , where the value at  $M_R$ ,  $(\Delta_{sol}^2)_{M_R}$  is a free parameter that we can choose as an input.

(b-2) The case where  $m_1^R > 0$  and  $m_2^R > 0$

In order to simplify the calculation and to see the essence of the analysis, we neglect the term  $s'^2 < 0.04$ . Thus we take  $a = b = 1$  and  $\cos \phi_1 = 1$ . Then, the submatrix relevant to  $\nu_1^R$  and  $\nu_2^R$  is given by

$$\tilde{m}'_\nu \simeq \begin{pmatrix} (1 + \Delta - \frac{2}{3}\epsilon) & \frac{\sqrt{2}}{3}\epsilon \\ \frac{\sqrt{2}}{3}\epsilon & 1 - \frac{1}{3}\epsilon \end{pmatrix} m_2^R. \quad (36)$$

After the diagonalization, we find

$$\begin{aligned} m_1 &= \left(1 + \frac{\Delta}{2} - \frac{1}{2}\epsilon + \text{sign}(\Delta) \frac{1}{2}\sqrt{D}\right) m_2^R, \\ m_2 &= \left(1 + \frac{\Delta}{2} - \frac{1}{2}\epsilon - \text{sign}(\Delta) \frac{1}{2}\sqrt{D}\right) m_2^R, \end{aligned} \quad (37)$$

where  $\text{sign}(\Delta)$  takes 1 for  $\Delta > 0$  and  $-1$  for  $\Delta < 0$  and

$$D = \left(\frac{1}{3}\epsilon - \Delta\right)^2 + \frac{8}{9}\epsilon^2. \quad (38)$$

The mass of the third one is  $m_3 = (1 - \epsilon)m_3^R$ . The mixing matrix is

$$\frac{1}{N} \begin{pmatrix} \text{sign}(\Delta)\sqrt{D} - (\frac{1}{3}\epsilon - \Delta) & -\frac{2\sqrt{2}}{3}\epsilon \\ \frac{2\sqrt{2}}{3}\epsilon & \text{sign}(\Delta)\sqrt{D} - (\frac{1}{3}\epsilon - \Delta) \end{pmatrix}, \quad (39)$$

where  $N$  is the normalization factor.

Now we multiply the above matrix to  $V_2$ . Aside from the unimportant phase and by taking  $c' \simeq 1$ , we find

$$\begin{aligned} V_{11} &\simeq \frac{1}{\sqrt{3}N} \left\{ \text{sign}(\Delta)\sqrt{D} - (\frac{1}{3}\epsilon - \Delta) + \frac{4}{3}\epsilon \right\}, \\ V_{12} &\simeq \frac{1}{\sqrt{3}N} \left\{ -\frac{2\sqrt{2}}{3}\epsilon + \sqrt{2} \left[ \text{sign}(\Delta)\sqrt{D} - (\frac{1}{3}\epsilon - \Delta) \right] \right\}. \end{aligned} \quad (40)$$

Now we find

$$\sin^2 2\theta_{sol} = \frac{8}{9} \left[ \frac{(\text{sign}(\Delta)\sqrt{D} + \Delta)^2 - \epsilon^2}{(\text{sign}(\Delta)\sqrt{D} + \Delta - \frac{\epsilon}{3})^2 + \frac{8}{9}\epsilon^2} \right]^2. \quad (41)$$

Firstly, since the mass matrix in Eq.(36) is real matrix, the CP violation phase  $\delta$  are stable and takes  $\delta = \pi/2$  at  $m_Z$ . Needless to say, the atmospheric neutrino mixing and  $s_{13}$  are stable.

(i) The stable  $\sin^2 2\theta_{sol}$

We focus on the solar neutrino mixing. From Eq.(41), we see that if  $|\Delta| \gg \epsilon$ ,  $\sin^2 2\theta_{sol} \simeq 8/9$ . For  $\Delta > 0$ , this condition is relaxed to the condition  $\Delta > 3\epsilon/2$ , where  $\sin^2 2\theta_{sol} \simeq 8/9$  is realized.

(ii) The unstable  $\sin^2 2\theta_{sol}$

Now we consider the case where  $\sin^2 2\theta_{sol}$  becomes small at  $m_Z$ . We observe that  $\sin^2 2\theta_{sol} \rightarrow 0$  as  $\Delta \rightarrow 0$ . This implies that  $\sin^2 2\theta_{sol}$  becomes small for  $\Delta \ll \epsilon$ , while it remains large value for  $\Delta > \epsilon$ .

Below, we examine the case  $\Delta \ll \epsilon$  to see the  $\Delta$  dependence of  $\sin^2 2\theta_{sol}$  in detail. We expand  $\sin^2 2\theta_{sol}$  in terms of  $\Delta/\epsilon$ . We obtain

$$\sin^2 2\theta_{sol} \simeq \frac{8}{9} \left( \frac{\Delta}{\epsilon} \right)^2. \quad (42)$$

The small angle which is consistent with the angle for the small angle MSW solution,  $\sin^2 2\theta_{sol} \simeq 10^{-2}$ , is realized if we take  $|\Delta| \sim \frac{1}{10}\epsilon$ . Next we examine the sign of  $(m_2^2 - m_1^2) \cos 2\theta_{sol}$ . For  $\Delta > 0$ , we find  $|V_{11}| \gg |V_{12}|$  at  $m_Z$ , i.e.,  $\cos 2\theta > 0$ , from Eq.(40). Then, as we can see from Eq.(37) with  $m_2^R \simeq m_2$ ,

$$m_1 - m_2 = m_2 \sqrt{\left(\frac{1}{3}\epsilon - \Delta\right)^2 + \frac{8}{9}\epsilon^2} > 0, \quad (43)$$

which means  $m_2^2 - m_1^2 < 0$ . Therefore we obtain  $(m_2^2 - m_1^2) \cos 2\theta_{sol} < 0$ . The same conclusion holds for  $\Delta < 0$  where  $|V_{11}| \ll |V_{12}|$  at  $m_Z$  ( $\cos 2\theta < 0$ ). That is, in both cases, we find  $(m_2^2 - m_1^2) \cos 2\theta_{sol} < 0$ . It is the standard lore that the small angle MSW solution is realized when  $(m_2^2 - m_1^2) \cos 2\theta_{sol} > 0$ , which is in conflict with our result. Recently, Gouvêa, Friedland and Murayama[11] have examined the case  $\cos 2\theta < 0$  (dark side) for  $m_2^2 - m_1^2 > 0$ . They found that the region  $\cos 2\theta \sim -0.2$  is still possible to explain the solar neutrino problem. However, this case corresponds to the large mixing case,  $\sin^2 2\theta \sim 0.96$  which is not our case. In conclusion, when  $\Delta \sim \frac{1}{10}\epsilon$ ,  $\sin^2 2\theta_{sol} \sim 0.01$  can be realized, but in this case the MSW mechanism does not work. Therefore, this case is not applicable to solve the solar neutrino problem.

## 4 Examples -Numerical analysis-

Since it is hard to search all parameter regions, we set  $m_2^0 = 0$  and then varied other parameters,  $\tilde{m}_1$ ,  $m_1^0$  and  $m_3^0$ . Here, we exhibit two examples, one for the stable case where the large angle MSW solution for the solar neutrino mixing is realized at  $m_Z$  and Dirac CP phase  $\sin \delta$  remain the maximal value, and the other for the case where  $\sin \delta$  becomes to be small at  $m_Z$ .

### (1) An example for the stable case

As an example, we adopted input values,  $(\tilde{m}_1, m_1^0, m_3^0) = (0.0699, -0.0117, -0.025)[\text{eV}]$  which give neutrino masses at  $M_R$  as  $(m_1^R, m_2^R, m_3^R) = (0.058200, 0.058509, -0.083509)[\text{eV}]$ . The values of observables at  $M_R$  and at  $m_Z$  are given in Table 1, for various values of  $\tan \beta$ .

Among various parameters, the parameters relevant to atmospheric neutrino mixings,  $\Delta_{atm}^2$  and  $\sin^2 2\theta_{atm}$ ,  $\sin \theta_{13}$  and  $\sin \delta$  are almost unchanged against the energy scale change for various values of  $\tan \beta$ . The scale dependence for  $\Delta_{sol}^2$  and  $\sin^2 2\theta_{sol}$  depend on the values of  $\tan \beta$ . From the data,

$$\begin{aligned} 0.5 < \sin^2 2\theta_{sol} < 1, \\ 1 \times 10^{-5}[\text{eV}^2] < \Delta_{sol}^2 < 1 \times 10^{-4}[\text{eV}^2], \end{aligned} \quad (44)$$

we have the restriction on  $\tan \beta$ ,

$$\tan \beta = 3 \sim 13, \quad (45)$$

which we can see from Table 1.

(2) An example to give a small Dirac CP phase at  $m_Z$

We took input values,  $(\tilde{m}_1, m_1^0, m_3^0) = (0.3, -0.59651, -0.007)[\text{eV}]$  where neutrino masses at  $M_R$  are  $(m_1^R, m_2^R, m_3^R) = (0.29651, 0.29652, -0.30352)[\text{eV}]$ . We show the values of observables at  $M_R$  and at  $m_Z$  in Table 2, for various values of  $\tan \beta$ .

As we can see from Table 2,  $\Delta_{atm}^2$ ,  $\sin^2 2\theta_{atm}$  and  $\sin^2 2\theta_{sol}$  are almost unchanged. On the other hand,  $\sin \theta_{13}$ ,  $\Delta_{sol}^2$  and  $\sin \delta$  change depending on  $\tan \beta$ . In particular,  $\sin \delta$  does not change much for small  $\tan \beta$ , while changes substantially for large  $\tan \beta$ . This result is consistent with the discussion given for the case  $m_1^R m_2^R < 0$  and  $\Delta \sim \epsilon/3$ .

In Fig.1 and Fig.2, we show the energy scale dependence of  $m_i^2 (i = 1, 2)$  and  $\sin \delta$  for  $\tan \beta = 4$  and 10, for the parameter set in Table 2. From Fig.1, we see that  $\Delta_{sol}^2$  increases as the energy scale becomes small and also as  $\tan \beta$  becomes large. In Fig.2, we see that  $\sin \delta$  decreases for both  $\tan \beta = 4$  and 10. However, much faster decrease is observed for the larger  $\tan \beta$ .

## 5 Summary and discussions

We considered the stability of the predictions by some special democratic-type neutrino mass matrix, which has the quite interesting intrinsic predictions as given in Eqs.(1) and (3). We assumed that this mass matrix is derived at the right-handed mass scale  $M_R$  by the see-saw mechanism[2], and then considered the mass matrix at the weak scale  $m_Z$  and its predictions by using the renormalization group.

We summarize the result as follows:

- The case (a): The fully hierarchical case

This is the case where the neutrino masses at  $M_R$  are either  $|m_3^R| \gg |m_1^R| \gg |m_2^R|$  or  $|m_3^R| \gg |m_2^R| \gg |m_1^R|$ . In this case, all predictions are stable and the predictions at  $M_R$  are valid at  $m_Z$ .

- The case (b): The hierarchical case with  $|m_1^R| \simeq |m_2^R|$

If  $m_1^R m_2^R < 0$ ,  $\sin^2 2\theta_{atm}$  and  $\sin^2 2\theta_{sol}$  are stable. The CP violation phase  $\sin \delta$  is also stable for  $\Delta \gg \epsilon$  or  $\Delta < 0$ . For  $\Delta \simeq \epsilon/3$ ,  $\sin \delta$  becomes unstable.

If  $m_1^R m_2^R > 0$ ,  $\sin^2 2\theta_{atm}$  and the CP violation phase  $\delta$  are stable. The solar mixing angle  $\sin^2 2\theta_{sol}$  is also stable for  $|\Delta| \gg \epsilon$ . For  $|\Delta| < \epsilon$ ,  $\sin^2 2\theta_{sol}$  becomes unstable. In particular, for  $\Delta \simeq \epsilon/10$ ,  $\sin^2 2\theta_{sol}$  at  $m_Z$  becomes small enough to be consistent with the mixing angle for the small angle MSW solution. However, this case does not realize the small angle MSW solution.

Our model based on the  $Z_3$  symmetry gives quite special predictions as given in Eqs.(1) and (3). We emphasize that our matrix is intrinsically complex matrix and contains the CP violation phase. In particular, our model predicts the maximal CP violation phase, which is in contrast to most of works where the real neutrino mass matrices are treated so that the prediction for the CP violation phase is out of reach. The prediction for the CP violation phase in the neutrino mass matrix will become a quite important topic in view of the near future projects to observe the neutrino oscillations, for example, in the neutrino factory.

It is our belief that  $Z_3$  symmetry is not only useful for describing the neutrino mass matrix, but also for the quark mass matrix. The work in this direction will be interesting, because we would like to embed the  $Z_3$  symmetry in the grand unification scheme.

**Acknowledgment** This work is supported in part by the Japanese Grant-in-Aid for Scientific Research of Ministry of Education, Science, Sports and Culture, No.12047218.

## Appendix A: Detailed derivations

(a) The standard parameterization of the mixing matrix

The particle data group[3] defines the mixing matrix as

$$V_{SF} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (\text{A.1})$$

(b) Diagonalization of  $m_\nu(M_R)$  in Eq.(10)

Here, we diagonalize the neutrino mass matrix at  $M_R$  and thus the predictions are given at  $M_R$ . In order to clarify the property of the democratic-type mass matrix, we first transform  $m_\nu(M_R)$  by the trimaximal matrix  $V_T$

$$V_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}, \quad (\text{A.2})$$

where  $\omega = e^{i2\pi/3}$  ( $\omega^3 = 1$ ) and the result is

$$V_T^T m_\nu(M_R) V_T = \begin{pmatrix} m_1^0 + \tilde{m}_1 & 0 & 0 \\ 0 & m_2^0 & \tilde{m}_1 \\ 0 & \tilde{m}_1 & m_3^0 \end{pmatrix}. \quad (\text{A.3})$$

Then, we transform further by

$$O_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (\text{A.4})$$

and we find

$$(V_T O_1)^T m_\nu(M_R) V_T O_1 = \begin{pmatrix} m_1^0 + \tilde{m}_1 & 0 & 0 \\ 0 & \tilde{m}_1 + m_2^0 + \Delta_- & \Delta_- \\ 0 & \Delta_- & -\tilde{m}_1 + m_2^0 + \Delta_- \end{pmatrix}. \quad (\text{A.5})$$

The matrix  $V_1 = V_T O_1$  is explicitly given by

$$V_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & i \end{pmatrix}. \quad (\text{A.6})$$



We have to transform further by the orthogonal matrix  $O_2$

$$O_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix}, \quad (\text{A.7})$$

where  $s'$  and  $c'$  are defined by Eq.(12). Now the mixing matrix is given by  $V = V_T O_1 O_2$  which is given in Eq.(11).

Below, we give some special cases.

(b-1) The  $m_3^0 = m_2^0$  case

We have  $s' = 0$  and  $c' = 1$  and the mixing matrix is now  $V = V_1$ . Then, the model predicts

$$\sin^2 2\theta_{atm} = 1, \quad \sin^2 2\theta_{sol} = \frac{8}{9}. \quad (\text{A.8})$$

There is no CP violation Dirac phase.

(b-2) The  $m_2^0 = 0$  case

The angle  $\theta'$  is determined by the ratio of  $m_2$  and  $m_3$ , and we have

$$\sin^2 2\theta_{sol} = \frac{4}{9} \frac{\beta + 2}{\beta}, \quad \sin^2 2\theta_{atm} = \frac{4}{9} \frac{(\beta + 1)(2\beta - 1)}{\beta^2}, \quad (\text{A.9})$$

where  $\beta = \sqrt{|m_2/m_3|} + \sqrt{|m_3/m_2|} \geq 2$ . If  $\beta$  is close to 2, we have the large solar neutrino mixing and also the large atmospheric neutrino mixing.

## References

- [1] K. Fukuura, T. Miura, E. Takasugi and M. Yoshimura, Phys. Rev. **D61** (2000) 073002.
- [2] T. Miura, E. Takasugi and M. Yoshimura, hep-ph/0003139.
- [3] C. Caso *et al.*, Eur. Phys. J. **C3** (1998) 1.
- [4] M. Apollonio *et al.*, Phys. Lett. **B466** (1999) 415.
- [5] S. M. Bilenky, J. Hošek and S. T. Petcov, Phys. Lett. **94B** (1980) 495;  
M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. **102B** (1981) 323.
- [6] V. Barger, S. Geer, R. Raja and K. Whisnant, hep-ph/0003184;  
V. Barger, S. Geer and K. Whisnant, Phys. Rev. **D61** (2000) 053004;  
Y. Kuno and Y. Mori, Talk at the ICFA/ECFA Workshop “Neutrino Factories based on Muon Storage Ring”, July 1999;  
Y. Kuno, in *Proceedings of the Workshop on High Intensity Secondary Beam with Phase Rotation*, September 1998.
- [7] N. Haba and N. Okamura, hep-ph/9906481.
- [8] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Nabarro, JHEP **9909** (1999) 015; Nucl. Phys. **B573** (2000) 652.
- [9] K. R. S. Balaji, A. S. Dighe, R. N. Mohapatra, M. K. Parida, hep-ph/0001310.
- [10] N. Haba, Y. Matui, N. Okamura, T. Suzuki, hep-ph/0005064.
- [11] A. de Gouvêa, A. Friedland and H. Murayama, Phys. Lett. **B490**(2000) 125.

Values at $M_R$ scale						
	$\sin \theta_{13}$	$\Delta_{atm}^2[\text{eV}^2]$	$\sin^2 2\theta_{atm}$	$\Delta_{sol}^2[\text{eV}^2]$	$\sin^2 2\theta_{sol}$	$\sin \delta$
	0.072148	$3.5865 \times 10^{-3}$	0.99997	$3.6048 \times 10^{-5}$	0.88195	1
Values at $m_Z$ scale						
$\tan \beta$	$\sin \theta_{13}$	$\Delta_{atm}^2[\text{eV}^2]$	$\sin^2 2\theta_{atm}$	$\Delta_{sol}^2[\text{eV}^2]$	$\sin^2 2\theta_{sol}$	$\sin \delta$
3	0.072149	$3.5837 \times 10^{-3}$	0.99998	$3.6422 \times 10^{-5}$	0.86307	1.0000
4	0.072150	$3.5818 \times 10^{-3}$	0.99999	$3.6702 \times 10^{-5}$	0.84934	1.0000
5	0.072151	$3.5793 \times 10^{-3}$	0.99999	$3.7084 \times 10^{-5}$	0.83116	1.0000
6	0.072152	$3.5763 \times 10^{-3}$	1.0000	$3.7584 \times 10^{-5}$	0.80832	1.0000
7	0.072153	$3.5728 \times 10^{-3}$	1.0000	$3.8218 \times 10^{-5}$	0.78068	1.0000
8	0.072155	$3.5688 \times 10^{-3}$	0.99999	$3.9006 \times 10^{-5}$	0.74831	1.0000
9	0.072156	$3.5643 \times 10^{-3}$	0.99999	$3.9968 \times 10^{-5}$	0.71149	1.0000
10	0.072158	$3.5594 \times 10^{-3}$	0.99998	$4.1123 \times 10^{-5}$	0.67078	1.0000
11	0.072160	$3.5540 \times 10^{-3}$	0.99996	$4.2491 \times 10^{-5}$	0.62696	1.0000
12	0.072162	$3.5481 \times 10^{-3}$	0.99993	$4.4087 \times 10^{-5}$	0.58102	1.0000
13	0.072164	$3.5418 \times 10^{-3}$	0.99988	$4.5926 \times 10^{-5}$	0.53404	1.0000
14	0.072166	$3.5351 \times 10^{-3}$	0.99982	$4.8021 \times 10^{-5}$	0.48711	1.0000
15	0.072168	$3.5280 \times 10^{-3}$	0.99974	$5.0380 \times 10^{-5}$	0.44124	1.0000

Table 1: The predicted values of various observable at  $m_Z$ . As input values at  $M_R$ , we choose  $(\tilde{m}_1, m_1^0, m_3^0) = (0.0699, -0.0117, -0.025)$  which are equivalent to the choice of neutrino masses at  $M_R$   $(m_1^R, m_2^R, m_3^R) = (0.058200, 0.058509, -0.083509)[\text{eV}]$ . In this case, observable are almost stable. However  $\sin^2 2\theta_{sol}$  becomes smaller as  $\tan \beta$  becomes larger.

Values at $M_R$						
	$\sin \theta_{13}$	$\Delta_{atm}^2 [\text{eV}^2]$	$\sin^2 2\theta_{atm}$	$\Delta_{sol}^2 [\text{eV}^2]$	$\sin^2 2\theta_{sol}$	$\sin \delta$
	0.004763	$4.2065 \times 10^{-3}$	1.0000	$6.1770 \times 10^{-6}$	0.88886	1
Values at $m_Z$						
$\tan \beta$	$\sin \theta_{13}$	$\Delta_{atm}^2 [\text{eV}^2]$	$\sin^2 2\theta_{atm}$	$\Delta_{sol}^2 [\text{eV}^2]$	$\sin^2 2\theta_{sol}$	$\sin \delta$
3	0.005870	$4.1939 \times 10^{-3}$	0.99997	$1.5964 \times 10^{-5}$	0.88887	0.82400
4	0.007538	$4.1851 \times 10^{-3}$	0.99993	$2.2724 \times 10^{-5}$	0.88886	0.65154
5	0.001015	$4.1740 \times 10^{-3}$	0.99984	$3.1305 \times 10^{-5}$	0.88881	0.49382
6	0.013674	$4.1606 \times 10^{-3}$	0.99968	$4.1619 \times 10^{-5}$	0.88871	0.37619
7	0.018061	$4.1451 \times 10^{-3}$	0.99941	$5.4560 \times 10^{-5}$	0.88853	0.29373
8	0.023293	$4.1275 \times 10^{-3}$	0.99900	$6.6999 \times 10^{-5}$	0.88825	0.23605
9	0.029372	$4.1080 \times 10^{-3}$	0.99839	$8.1782 \times 10^{-5}$	0.88783	0.19496
10	0.036315	$4.0869 \times 10^{-3}$	0.99755	$9.7732 \times 10^{-5}$	0.88722	0.16505
11	0.044144	$4.0643 \times 10^{-3}$	0.99640	$1.1464 \times 10^{-4}$	0.88639	0.14282

Table 2: The predicted values of various observable at  $m_Z$ . As input values at  $M_R$ , we choose  $(\tilde{m}_1, m_1^0, m_3^0) = (0.3, -0.59651, -0.007)$  which are equivalent to the choice of neutrino masses at  $M_R$   $(m_1^R, m_2^R, m_3^R) = (-0.29651, 0.29652, -0.30352)[\text{eV}]$ . Note that relative sign of  $m_1^R$  and  $m_2^R$  is minus. In this case,  $\sin \delta$  becomes to be small.

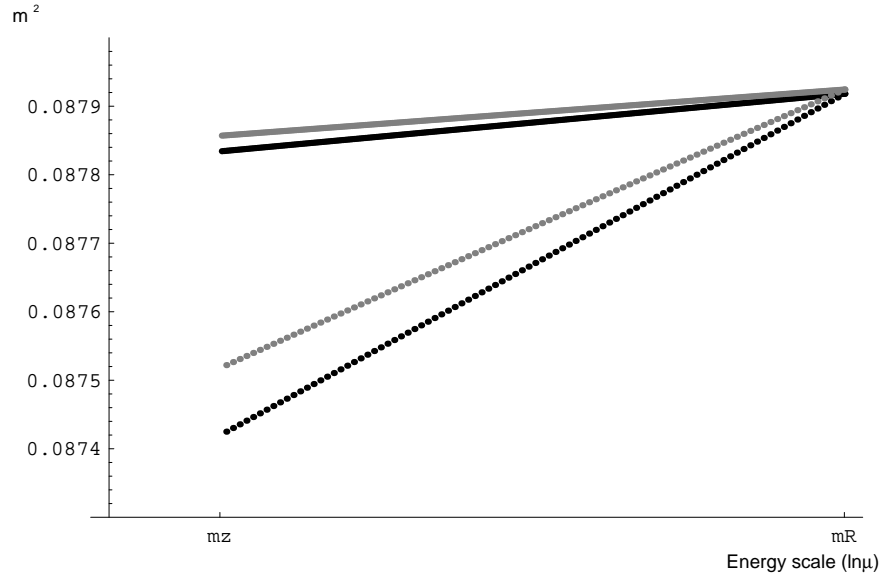


Figure 1: Energy scale dependence of 1st and 2nd generation's neutrino masses  $[\text{eV}^2]$ (squared masses) for the parameter set at  $M_R$  given in Table 2 ( $m_1^R \cdot m_2^R < 0$ ). Black line (dots) is for  $m_1^2$  and gray line (dots) is for  $m_2^2$ . Solid line is for  $\tan \beta = 4$  and dashed line is for  $\tan \beta = 10$ . The horizontal axes describe energy scale( $\log \mu$ ).

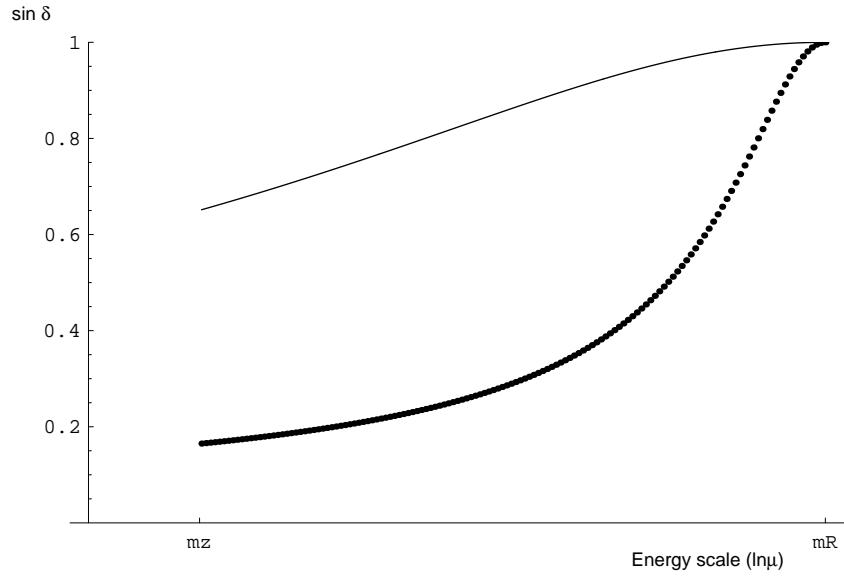


Figure 2: Energy dependence of  $\sin \delta$  for the parameter set at  $M_R$  given in Table 2 ( $m_1^R \cdot m_2^R < 0$ ) and this case shows small CP violation angle at  $m_Z$ , while it is large at  $M_R$ . Solid curves correspond to  $\tan \beta = 4$  while dashed are for  $\tan \beta = 10$ .